Global Optimization of Electromagnetic Devices using Quantum Particle Swarm Optimization with Novel Methodology

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Quantum Particle Swarm Optimization (QPSO) is a population-based swarm intelligence algorithm, and has been successfully applied to solve a wide scope of optimization design problems. However, it suffers from premature convergence and insufficient diversity at the later stages of the evolution. To address this problem in this work, a novel methodology is employed to choose the fittest particle and a new mutation mechanism is introduced in which a mutation operation is exerted on the global best particle to avoid the population from clustering and facilitating the particle to escape the sub-optimum more easily. Also, some parameter updating strategy is proposed to facilitate the algorithm to keep a good balance between exploration and exploitation searches. The numerical results on three case study are reported to validate the proposed methodology.

Index Terms— Electromagnetic design problem, global optimization, mutation, quantum mechanics.

I. INTRODUCTION

The optimization design of electromagnetic devices has been a particular emphasis in the field of electromagnetic optimization specifically with the expansion of modern technology and the development of numerical analyzing machinery.

In last few years, the evolutionary algorithms, such as tabu search method, ant colony optimization, genetic algorithm and simulated annealing method, have become very popular in the optimization society and have successfully applied to a wide scope of electromagnetic design optimizations. In contrast to traditional single point algorithms, evolutionary algorithms are population based stochastic methods which are characterized by the capability to determine the global optimal solution in a very short period, especially when the objective functions are not deterministic. However, there is no such global stochastic method that can be applied equally and successfully to all electromagnetic optimization problems. Consequently, it is important to investigate new universal optimizers in the study of electromagnetic optimization problems.

Particle swarm optimization is a recent entrant to the world of evolutionary algorithms. It was originated by Kennedy and Eberhart based on metaphor of the social behavior of birds flocking and fish schooling in their search for food [1]. PSO algorithm is very simple in concepts and easy in implementation. Nevertheless, as compared to its other well developed stochastic counterpart, PSO method is an emerging methodology and is still in its infancy phase. For example, the PSO method encounters to a premature convergence while searching for global optima of a hard optimization problems. The stagnation phenomena is also likely to occur in PSO that results the algorithm to be trapped to local minima. Thus, to address such issues in PSO, a quantum based version of particle swarm optimization (QPSO) was proposed in [2]. In QPSO, the activities of the particles follows the principle of quantum mechanics, in contrary to the principle of Newtonian mechanics being supposed in PSO. Hence, instead of the Newtonian random walk some sort of quantum motion is enforced in the search process of QPSO to guarantee that the particles can appear in any position and keep a balance between local and global searches. However, there are still many issues in QPSO which should be addressed.

In this context, a novel methodology is used for the selection of the personal best particle and also a new mutation method is applied to the global best particle, in addition a new parameter updating rule is proposed in an improved version of QPSO as reported in this work to further enhance the convergence process and intensify the performance of the QPSO algorithm.

II. PROPOSED QPSO METHOD

A. Selection of the Fittest Particle

In the QPSO algorithm the diversity of the population is high at the initial stage of the evolution process. However, with the progress of the search process the convergence of the particles makes the diversity be declining rapidly, that boosts the local search capability (exploitation) but weaken the global search capability (exploration) of the algorithm. When the diversity of the particle swarm becomes low after middle or later stage of the search process then the particles may converge into a small area that makes the further search impossible. At that time, if the particle with global best position is at local minima or sub optima then premature convergence occurs.

Thus, to avoid such difficulty and to improve the algorithm performance, some modification is made in QPSO as detailed in the follows.

Firstly, a new particle is generated in the current search domain by using the following methodology:

\[ p_{b2}(t) = x(t) - \frac{f(p_g(t))}{f(p_{best}(t))} \]  \( (1) \)

where \( f \) represent the fitness of the particle in issue, \( p_{best} \) and \( p_g \) are the particles with the personal and global best positions respectively, \( x(t) \) is the position vector of the current iteration.
One then compares the \(p_{b1}\) particle with the personal best particle in the current population. If the \(p_{b2}\) particle has better fitness than the \(p_{b1}\) particle, the \(p_{b2}\) will be replaced by the \(p_{b1}\) one; otherwise, the \(p_{b1}\) particle is persisted in the same position for the generation of next cycle in the updating process.

The mean best position is the average position of the personal best particles. So, when the diversity is low then this methodology will select a fittest personal best particle that will affect the mean best position and will reinitialize it. The reason for re-initialization of the mean best position is that when the diversity is low, then the distance between the mean best and current particle’s is very small for the particle to escape the local minima. Thus, re-initializing the mean best position will enlarge the distance between the mean best and the current particle that will make particles explode temporarily.

### B. Mutation Mechanism

Secondly, a new mutation mechanism is introduced by exerting the following mutation operation on the particle with the global best position:

\[
p_g^\prime = z \cdot p_g \times (1 + E_t)
\]  

(2)

where \(E_t\) is a random number generated using exponential probability distribution and \(z\) is a user defined constant which is set to be 0.001 using an error and trail method after comprehensive experiments on a wealth of case studies.

When the proposed mutation strategy is applied, the displacement of the global best particle will make increase the distance between the personal best particles and the mean best position and thus, increases the diversity. In this context, the displaced global best particle will be pulled away the mean best from its original position which will increases the distance between the mean best position and the current particles that will enhance the search scope and in this way the algorithm will avoid being trapped to local minima and consequently, would achieve a better performance of the algorithm.

### C. Parameter Updating Strategy

The contraction expansion coefficient \(\alpha\) is an important parameter to control the convergence speed of the algorithm. Hence, without adjusting the value of \(\alpha\) will leads to an improper balance between exploration and exploitation searches. Therefore, different researchers have proposed different strategies to adjust the \(\alpha\) parameter to enhance the convergence speed of the algorithm.

Thus, in this work to tradeoff between the exploration and exploitation searches and to intensify the convergence speed of the algorithm a new parameter updating formulae is proposed as:

\[
\alpha = (1 - 0.5) \times (1 - t / \text{Maxiter})^9
\]

(3)

\[
q = \exp(t - \text{rand})
\]

(4)

where \(\text{rand}\) is a uniform random number within interval \([0, 1]\), \(t\) is the current iteration and \(\text{Maxiter}\) is the maximum number of iterations.

## III. Numerical Results and Conclusions

To validate the effectiveness and the global search capability of the proposed QPSO, it has been applied to two standard mathematical test functions and an electromagnetic design problem [3]. The proposed QPSO is compared with the original QPSO [2], GQPSO [4] (Gaussian Quantum Behaved Particle Swarm Optimization approaches for constrained engineering design problems) and LIQPSO [5] (An Improved Quantum behaved Particle Swarm Optimization Algorithm based on Linear Interpolation). Moreover,

\[
f_1(x) = \frac{1}{4000} \sum_{i=1}^{n} x_i^2 - \frac{1}{4} \prod_{i=1}^{n} \cos \left(\frac{x_i}{\sqrt{i}}\right) + 1 \left(x_i, e^{[-600, 600]}\right)
\]

(5)

\[
f_2(x) = \frac{d}{i=1} \left| x_i + \prod_{i=1}^{d} \left| x_i \right| \right| \left(x_i, e^{[-100, 100]}\right)
\]

(6)

All these functions are minimization problems and the minimum value for each objective function is zero. Table I and Table II tabulate the average performance comparison of different optimal algorithms.

It can be analysed from the results in Table I and Table II that the proposed QPSO method can escape from the large number of local minima and converge to the global optimum by using a relatively less number of generations. Nevertheless, the proposed QPSO is a global optimizer and has outperformed all other tested optimal algorithms in terms of both the solution quality (objective function values) and convergence speed (number of iterations).

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<table>
<thead>
<tr>
<th>Function</th>
<th>QPSO</th>
<th>GQPSO</th>
<th>LIQPSO</th>
<th>Proposed</th>
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</thead>
<tbody>
<tr>
<td>(f_1(x))</td>
<td>8.2689×10^9</td>
<td>1.1836×10^7</td>
<td>8.1478×10^6</td>
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<td>(f_2(x))</td>
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<td>1.3401×10^10</td>
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</tbody>
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### Table II

<table>
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<tr>
<th>Algorithms</th>
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<th>Mean</th>
<th>SD</th>
<th>Max</th>
<th>(r_2)</th>
<th>h/2</th>
<th>d/2</th>
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<tr>
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<td>0.0959</td>
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<tr>
<td>Proposed</td>
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<td>0.1012</td>
<td>3.1144</td>
<td>0.2769</td>
<td>0.3422</td>
</tr>
</tbody>
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### REFERENCES


